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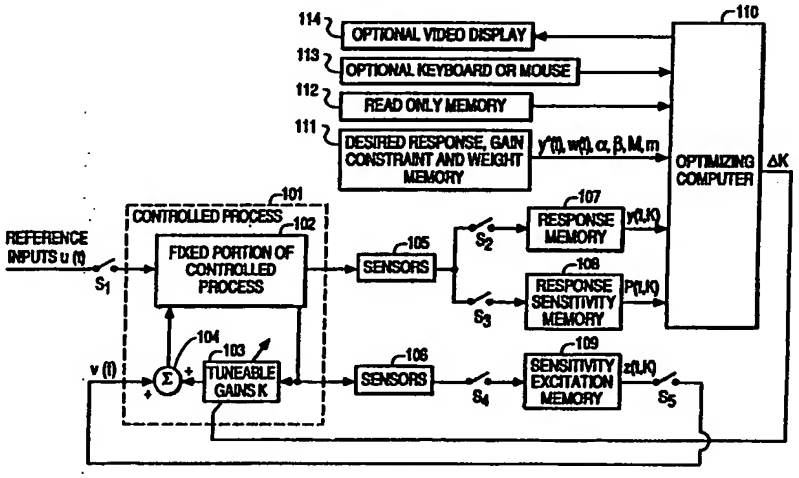

(57) Abstract

A method for shaping the time responses of a controlled physical process comprises the step of tuning selected gains of the process for shaping the time responses output from the process. In particular, additional inputs, outputs, and sensors (105, 106) are added to the controlled physical process (101) and are connected to memory elements (107-109) and to an optimizing computer (110) through data-path switches (S_1-S_5). A series of experiments generates the process time responses and the response sensitivities for storage in memory. Desired process time responses are created and stored in read only memory (112) of associated apparatus. An objective function is formed as a summation of the two-norms of weighted process time response errors. The response sensitivities are used to form a quadratic approximation of the objective function. The optimizing computer is used to minimize this approximate objective function subject to gain constraints. The new gain values are used in the controlled physical process to obtain the new process time responses. The desired process time responses, weightings, and gain constraints are modified and new approximations of the objective function are formed and minimized until process time responses are achieved that satisfy the design requirements.

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METHOD AND APPARATUS FOR SHAPING THE TIME RESPONSES OF A CONTROLLED PROCESS

FIELD OF THE INVENTION

The present invention relates to a method for tuning selected gains of a controlled process to shape the time responses output of the process, and more specifically to the generation and utilization of process response sensitivities and a quadratic optimization method to iteratively select gain values that result in improved process time responses.

BACKGROUND OF THE INVENTION

The ability to precisely shape the time responses of a highly complex multi-input, multi-output controlled process is of considerable practical interest to all areas of industrial, military, and academic controls. Most control systems design methods rely upon a mathematical model of the process. The use of differential equations to form an accurate model of a complex process is usually very difficult and time-consuming. Furthermore, the practical qualities of the design are extremely dependant upon the fidelity of the model. What is needed is a powerful and intuitive multi-output design approach that can reliably shape the time responses of the physical process without the aid of a mathematical model.

Output response sensitivity information has been used in the analysis and tuning of control systems since the 1960's. Reference is made to "Method of Sensitivity Points in the Investigation and Optimization of Linear Control Systems" by Petar V. Kokotovic, 1964. These sensitivity signals may be obtained from the physical process, a mathematical model, or a combination of both. The prior art includes the method of Variable Components for the generation of the response sensitivity signals in the manner described in accordance with the present invention. Reference is made to pp. 5-13 of "Sensitivity Methods and Slow Adaptation" by Douglas Scott Rhode, 1990. The prior art also forms a simple least squares problem from the process time response, the response sensitivities, and the desired time response for the purpose of finding gain values which will minimize the rms error between the tuned process time response and the desired response. Reference is made to pp. 7-10 of "Multi-Input / Multi-Output Sensitivity Points Tuning" by Stephen Think Hung, 1985.

A straight least squares solution is often too simplistic to adequately shape the time responses of highly complex multi-output controlled processes. Performance tradeoffs imposed by the physics of the controlled process can not be adequately explored, and the case of multiple solutions that would result in similar process responses can not be properly addressed.

SUMMARY OF THE INVENTION

The present invention resolves these shortcomings of the prior art. It describes an apparatus for the automated generation of the sensitivity signals and an extremely powerful optimization method for the selection of gain values that will improve the quality of the multi-output process responses. The present method and apparatus should find application in a wide diversity of physical processes to be controlled. One exemplary application may be in the real-time, automatic tuning of an audio equalizer that comprises part of an audio reproduction system. The present invention provides a means for tuning the equalizer in a manner so as to compensate for the acoustic imperfections present in the amplifier, speakers, and listening environment.

To this end, the present invention comprises a method for shaping the time responses of a multi-input, multi-output controlled physical process. The method comprises the steps of storing desired physical process responses in read only memory, generating a series of experiments on the physical process to generate current physical process responses, storing the current physical process responses in random access memory, generating and storing response sensitivities for the current physical process responses by the method of Variable Components, calculating new gain values by quadratic optimization, predicting new process responses, and applying the new gain values to the controlled physical process.

Apparatus according to the present invention comprises data path switches for gating data inputs to memory associated with an optimizing computer. A display associated with the computer provides a plurality of windows for looking into the controlled physical process. By means of a mouse of the computer, the process windows may be dynamically reconfigured and automatically display a plurality of parameters of the controlled process.

BRIEF DESCRIPTION OF THE DRAWINGS

In order that the invention may be more clearly understood, reference will now be made to the accompanying drawings, in which an embodiment of the invention is shown for purposes of illustration, and in which:

FIG. 1 shows a block diagram representation of a system for shaping the time responses of a controlled process in accordance with the invention, which includes the controlled process, process output sensors, data-path switches, memory elements, optimizing computer, keyboard or mouse, and video display.

FIG. 2 shows a block diagram representation of the mathematical algorithm that is implemented in the optimizing computer for the selection of the gain adjustments and the

calculation of the predicted new process time responses in accordance with the invention.

FIG. 3 shows a flowchart describing a flow of steps to shape the time responses of a controlled process in accordance with the invention.

FIG. 4 illustrates a time response window which is a component of the quadratic optimization method in accordance with the invention, and which shows a current process time response, desired response, predicted new response, and weights.

FIG. 5 illustrates a gain space window which is a component of the quadratic optimization method in accordance with the invention, and which shows a two-dimensional plane in the gain space along with gain constraints and weights.

FIG. 6 illustrates a gain space window which shows the tuning history of a gain along with a gain constraint and weight.

FIG. 7 illustrates a gain space window which shows a two-dimensional plane in the gain space along with contours of constant value of the objective function that is formed during the optimization process in accordance with the invention.

FIG. 8 illustrates a gain space window which shows a direction in the gain space along with the values of the gains and the value of the objective function along this direction.

FIG. 9 illustrates a gain space window which shows a direction in the gain space along with the values of the gains and the value of the objective function along this direction.

FIG. 10 illustrates a computer display which aids in the quadratic optimization procedure in accordance with the invention, and which includes two time response windows and five gain space windows.

DETAILED DESCRIPTION OF THE INVENTION

An embodiment of a system for shaping the time responses of a controlled process according to the invention will be described, referring to the accompanying drawings.

FIG. 1 shows a block diagram of an embodiment of the invention. Controlled process **101** includes a fixed portion **102**, tuneable gains **103**, and summing junction **104**. Summing junction **104** adds inputs v(t) to outputs of tuneable gains **103** and feeds the sum to fixed portion **102**. One set of outputs of fixed portion **102** feeds sensors **105**, and the other set of outputs feeds sensors **106** and tuneable gains **103**. Switch S_1 admits reference inputs u(t) to fixed portion of controlled process **102**. These reference inputs may be composed of command signals as well as disturbances. Switch S_2 admits the output of sensors **105** to response memory **107**. Switch S_3 admits the output of sensors **105** to response sensitivity memory **108**. Switch S_4 admits the output of sensors **106** to sensitivity excitation memory

109. Switch S_5 admits the output of sensitivity excitation memory 109 to summing junction 104 as input v(t). Optimizing computer 110 can read the contents of response memory 107 and response sensitivity memory 109. Optimizing computer 110 can read and write to the contents of desired response, gain constraint and weight memory 111. Optimizing computer 110 can adjust tuneable gains 103. Optimizing computer 110 receives execution instructions from read-only memory 112. Optimizing computer 110 receives commands from optional keyboard or mouse 113. Optimizing computer 110 sends information to optional video display 114.

All outputs of controlled process 101 are digitally sampled and conditioned by sensors 105 and sensors 106. Controlled process 101 produces time responses y(t, K) when excited by reference inputs u(t). The additional inputs and outputs are added to a controlled process 101 that is to be tuned in accordance with the invention.

Experiments generate and record in memory the process time responses and response sensitivities by the method of Variable Components. An initial experiment is run with data-path switches S_1, S_2, and S_4 closed and switches S_3 and S_5 open. Controlled process 101 is excited by reference input u(t), and all inputs v(t) are set to zero. The process time responses y(t, K) are sensed by sensors 105 and recorded in response memory 107. The sensitivity excitation signals z(t, K) are sensed by sensors 106 and recorded in sensitivity excitation memory 109.

A subsequent set of experiments are run with data-path switches S_1, S_2, and S_4 open and switches S_3 and S_5 closed. Controlled process 101 is not excited by reference input u(t). The jth experiment of the set is run with controlled process 101 input $v_j(t)$ equal to the sensitivity excitation signal $z_j(t, K)$ recorded in sensitivity excitation memory 109 during the initial experiment. All other inputs v(t) are set to zero. The controlled process 101 outputs sensed by sensor 105 during the jth experiment are the first partial derivatives (sensitivities) of the process time responses with respect to the tuneable gain K_j, and are recorded in response sensitivity memory 109. Reference is made to pp. 5-13 of "Sensitivity Methods and Slow Adaptation" by Douglas Scott Rhode, 1990.

Optimizing computer 110 utilizes information stored in response memory 107, response sensitivity memory 108, and desired response, gain constraint and weight memory 111to obtain the gain adjustments ΔK that will modify the tuneable gains 103. FIG. 2 shows a block diagram of the calculations that are executed in optimizing computer 110 to produce gain adjustments ΔK and predicted new process time responses $\hat{y}(t, K, \Delta K)$. The instructions necessary for the calculation of the gain adjustments and predicted new process time responses

are contained in read-only memory **112** of FIG. **1**. Each computational block in FIG. **2** is labeled with the mathematical operations that form that block's output quantities. Computational block **201** produces matrix Q(K) and vector b(K) through the addition and multiplication of the process responses y(t, K), response sensitivities P(t, K), desired responses $y^*(t)$, response weights w(t), and response weights α. Computational block **202** produces matrix Q_S, vector $b_S(K)$, matrix M_H, and vector $\bar{m}(K)$ through the addition and multiplication of the gain constraints M, gain constraints m, and constraint weights β. Computational block **203** produces matrices V_{H0} and M_H^+ through the singular value decomposition of the matrix M_H provided by block **202**. Computational block **204** produces vector ΔK_{min} through the multiplication of matrix M_H^+ provided by block **203** and vector $\bar{m}(K)$ provided by block **202**. Computational block **205** produces matrix $Q_E(K)$ and vector $b_E(K)$ through the addition and multiplication of the outputs provided by blocks **201** through **204**. Computational block **206** produces matrix $Q_E^+(K)$ through the spectral decomposition of the matrix $Q_E(K)$ provided by block **205**. Computational block **207** produces vector w through the multiplication of matrix $Q_E^+(K)$ provided by block **206** and vector $b_E(K)$ provided by block **205**. Computational block **208** produces the vector of gain adjustments ΔK through the addition and multiplication of the outputs provided by blocks **203**, **204**, and **205**. Computational block **209** produces the predicted new process responses $\hat{y}(t, K, \Delta K)$ through the addition and multiplication of the vector of gain adjustments ΔK provided by block **208**, the process responses y(t, K), and the response sensitivities P(t, K).

The computational procedure for obtaining the gain adjustments ΔK and predicted new process time responses $\hat{y}(t, K, \Delta K)$ will now be explained in greater detail. Vector notation is used to represent the tuneable gains **103**,

$$K := [K_1, ..., K_{nk}]^T, \quad (1)$$

and the gain adjustments produced by optimizing computer **110**,

$$\Delta K := [\Delta K_1, ..., \Delta K_{nk}]^T. \quad (2)$$

The superscript T denotes the transpose of a vector or matrix. The notation : = means that the quantity on the left side of the equation is defined by the expression on the right. The subscript nk is the total number of tuneable gains.

Time responses $y^*(t)$ are desired of the process when it is excited by reference inputs u(t). Form a time response error for each output of the fixed portion of controlled process **102**

$$e_i(t, K) := y_i^*(t) - y_i(t, K), \quad (3)$$

and multiply each time response error by a weighting $w_i(t)$ to form a weighted time response error

$$e_{Wi}(t, K) := w_i(t)\, e_i(t, K). \tag{4}$$

Quantities such as $y_i(t, K)$, $y_i^*(t)$, and $w_i(t)$ are vectors recorded in memory. When the subscript i is omitted, as in $y(t, K)$, the quantity refers to all of the vectors in the set.

The objective function is

$$r(K) := \sum_{i=1}^{ny} \alpha_i^2 \, || e_{Wi}(t, K) ||^2 , \tag{5}$$

where each α_i is a weighting scalar and ny is the total number of outputs of the fixed portion of controlled process **102**. The two-norm of a vector v of length nv is calculated as

$$|| v || \equiv \left(\sum_{i=1}^{nv} v_i^2 \right)^{\frac{1}{2}} = (v^T v)^{\frac{1}{2}} . \tag{6}$$

The selection of gains K that minimize the objective function (5) results in the shaping of time responses of controlled process **101** to resemble the desired time responses. There are gain constraint functions that restrict the values achievable by the gains K,

$$d_j(K) := || M_j K - m_j ||^2 , \tag{7}$$

where each M_j is a constant matrix and each m_j is a constant vector. The objective and constraint functions are nonnegative definite, scalar, real-valued functions of the gains K. The constrained minimization problem is

$$\begin{aligned} &\text{Find K to minimize} && r(K) \\ &\text{subject to} && d_j(K) \le \delta_j^2, \quad j = 1, 2, \ldots, nd \end{aligned} \tag{8}$$

where each δ_j is a nonnegative scalar and nd is the total number of gain constraints.

A nominal gain solution K is chosen. A quadratic approximation $\hat{r}(K, \Delta K)$ of the objective function is formed about the nominal solution. Gain adjustments ΔK are found that minimize $\hat{r}(K, \Delta K)$ subject to the gain constraints. The tuneable gains **103** are adjusted to the new values $K+\Delta K$ and new time responses are generated from controlled process **101**. If the new process time responses do not meet the design requirements, then new response sensitivities are generated and a new quadratic approximation of r (K) is formed and minimized.

The first partial derivatives (sensitivities) of the time responses $y_i(t, K)$ with respect to each tuneable gain K_j are used to form the quadratic approximation of the objective function. The time response sensitivities of each output of fixed portion of controlled process **102** are gathered to form the quantity

$$P_i(t, K) := \left[\frac{\partial y_i(t, K)}{\partial K_1} \quad \cdots \quad \frac{\partial y_i(t, K)}{\partial K_{nk}} \right]. \tag{9}$$

The quantity $P_i(t, K)$ is a matrix whose columns are the time response sensitivity vectors.

The shape of the new time response $y_i(t, K+\Delta K)$ from each output of fixed portion of controlled process **102** may be predicted for small gain adjustments ΔK by the first-order Taylor's series approximation

$$\hat{y}_i(t, K, \Delta K) := y_i(t, K) + P_i(t, K)\,\Delta K, \tag{10}$$

where the hat differentiates the predicted new time response $\hat{y}_i(t, K, \Delta K)$ from the actual new time response $y_i(t, K+\Delta K)$. The approximate time response error for each output is

$$\begin{aligned}\hat{e}_i(t, K, \Delta K) &:= y_i^*(t) - \hat{y}_i(t, K, \Delta K) \\ &= y_i^*(t) - y_i(t, K) - P_i(t, K)\,\Delta K \\ &= e_i(t, K) - P_i(t, K)\,\Delta K. \end{aligned} \tag{11}$$

Each weighted approximate time response error is

$$\hat{e}_{Wi}(t, K, \Delta K) := w_i(t)\,\hat{e}_i(t, K, \Delta K), \tag{12}$$

and the quadratic approximation of the objective function r (K) is

$$\hat{r}(K, \Delta K) := \sum_{i=1}^{ny} \alpha_i^2 \left\| \hat{e}_{Wi}(t, K, \Delta K) \right\|^2. \tag{13}$$

This approximation may also be expressed as

$$\hat{r}(K, \Delta K) = \sum_{i=1}^{ny} \alpha_i^2 \left\| e_{Wi}(t, K) - P_{Wi}(t, K)\,\Delta K \right\|^2, \tag{14}$$

where each weighted matrix of time response sensitivities is given by

$$P_{Wi}(t, K) := w_i(t)\,P_i(t, K). \tag{15}$$

The quadratic form of the quadratic approximation of the objective function is

$$\hat{r}(K, \Delta K) = [\Delta K]^T Q(K) [\Delta K] - 2[\Delta K]^T b(K) + r(K), \tag{16}$$

where

$$Q(K) := \sum_{i=1}^{ny} \alpha_i^2\, P_{Wi}^T(t, K)\, P_{Wi}(t, K), \tag{17}$$

and

$$b(K) := \sum_{i=1}^{ny} \alpha_i^2\, P_{Wi}^T(t, K)\, e_{Wi}(t, K). \tag{18}$$

Equations 3, 4, 15, 17, and 18 are implemented in computational block **201** of FIG. **2** to calculate the nk-by-nk matrix Q(K) and the nk-by-one vector b(K).

Rewrite the constraint functions of (7) as explicit functions of both K and ΔK.

$$d_j(K, \Delta K) := \left\| M_j \Delta K - \bar{m}_j(K) \right\|^2, \tag{19}$$

where the vectors $\bar{m}_j(K)$ are

$$\bar{m}_j(K) := m_j - M_j K. \tag{20}$$

A distinction must be made between those constraints in (8) for which δ_j is equal to zero, and

those for which δ_j is strictly greater than zero. Since the two-norm function is nonnegative definite, the constraint

$$\left|\left| M_j \Delta K - \overline{m}_j(K) \right|\right|^2 \leq 0 \tag{21}$$

can be satisfied if and only if

$$M_j \Delta K = \overline{m}_j(K) . \tag{22}$$

Arrange the constraints so that

$$\delta_j > 0 , \text{ for all } j = 1, ..., p \tag{23}$$

and

$$\delta_j = 0 , \text{ for all } j = p+1, ..., nd. \tag{24}$$

The constraints for which δ_j is strictly greater than zero are called soft constraints, and the constraints for which δ_j is equal to zero are called hard constraints.

Define the soft constraint objective function to be a linear combination of the soft constraint functions,

$$r_S(K, \Delta K) := \sum_{j=1}^{p} \beta_j^2 \, d_j(K, \Delta K) = \sum_{j=1}^{p} \beta_j^2 \left|\left| M_j \Delta K - \overline{m}_j(K) \right|\right|^2 , \tag{25}$$

where each β_j is a scalar weighting just like each scalar α_i in the objective function. The quadratic form of the soft constraint objective function is given by

$$r_S(K, \Delta K) = [\Delta K]^T Q_S(K) [\Delta K] - 2[\Delta K]^T b_S(K) + c_S(K) , \tag{26}$$

where

$$Q_S := \sum_{j=1}^{p} \beta_j^2 M_j^T M_j , \tag{27}$$

$$b_S(K) := \sum_{j=1}^{p} \beta_j^2 M_j^T \overline{m}_j(K) , \tag{28}$$

$$c_S(K) := \sum_{j=1}^{p} \beta_j^2 \overline{m}_j^T(K) \overline{m}_j(K) . \tag{29}$$

Equations 20, 27, and 28 are implemented in computational block 202 of FIG. 2 to calculate the nk-by-nk matrix Q_S and the nk-by-one vector $b_S(K)$.

Define an extended quadratic approximation of the objective function to be the original approximation plus the soft constraint objective function,

$$\hat{r}_E(K, \Delta K) := \hat{r}(K, \Delta K) + r_S(K, \Delta K) . \tag{30}$$

This extended function may also be expressed as

$$\hat{r}_E(K, \Delta K) = \sum_{i=1}^{ny} \alpha_i^2 \left\| P_{Wi}(t, K)\, \Delta K - e_{Wi}(t, K) \right\|^2 + \sum_{j=1}^{p} \beta_j^2 \left\| M_j\, \Delta K - \bar{m}_j(K) \right\|^2 . \tag{31}$$

During optimization, the value of each β_j may be increased until the corresponding soft constraint is satisfied. The quadratic form of the extended quadratic approximation is

$$\hat{r}_E(K, \Delta K) = [\Delta K]^T (Q(K) + Q_S)[\Delta K] - 2[\Delta K]^T (b(K) + b_S(K)) + (r(K) + c_S(K)) . \tag{32}$$

Stack all the hard constraints to form a matrix M_H and a vector $\bar{m}_H(K)$ such that

$$M_H := \begin{bmatrix} M_{p+1} \\ \vdots \\ M_{nd} \\ 0 \end{bmatrix}, \quad \text{and} \quad \bar{m}_H(K) := \begin{bmatrix} \bar{m}_{p+1}(K) \\ \vdots \\ \bar{m}_{nd}(K) \\ 0 \end{bmatrix} \tag{33}$$

where the lower block of M_H is a zero matrix of the correct dimensions to make M_H nk-by-nk square, and the lower block of $\bar{m}_H(K)$ is a zero vector of the correct length to make $\bar{m}_H(K)$ have nk rows. Equations 33 are implemented in computational block 202 of FIG. 2 to calculate the nk-by-nk matrix M_H and the nk-by-one vector $\bar{m}_H(K)$. All the hard constraints are satisfied if the hard constraint equation

$$M_H\, \Delta K = \bar{m}_H(K) \tag{34}$$

is satisfied. The complete set of gain adjustment vectors ΔK that satisfy (34) must be found. An expression for this set may be found through the singular value decomposition of the square matrix M_H. The SVD of M_H is given by

$$M_H = U_H \Sigma_H V_H^T , \tag{35}$$

where Σ_H is an nk-by-nk diagonal matrix of the singular values of M_H arranged in decreasing order, and U_H and V_H are both nk-by-nk orthogonal matrices. Partition these matrices such that

$$U_H = [\, U_{H1} \;\; U_{H0} \,], \quad \Sigma_H = \begin{bmatrix} \Sigma_{H1} & 0 \\ 0 & 0 \end{bmatrix}, \quad V_H = [\, V_{H1} \;\; V_{H0} \,], \tag{36}$$

where Σ_{H1} is a diagonal matrix of the strictly positive singular values o. M_H. The pseudoinverse of M_H is calculated as

$$M_H^+ = V_{H1} \Sigma_{H1}^{-1} U_{H1}^T , \tag{37}$$

and the columns of V_{H0} form an orthonormal basis for the null space of M_H. Let nw be the number of columns of the matrix V_{H0}. The singular value decomposition of the matrix M_H along with equations 36 and 37 are implemented in computational block 203 of FIG. 2 to calculate the nk-by-nk matrix M_H^+ and the nk-by-nw matrix V_{H0}. The minimum two-norm solution to the hard constraint equation (34) is

$$\Delta K_{min} := M_H^{+}\, \bar{m}_H(K) . \tag{38}$$

Equation 38 is implemented in computational block 204 of FIG. 2 to calculate the nk-by-one vector ΔK_{min}. The complete solution to (34) is given by

$$\Delta K = \Delta K_{min} + V_{H0}\, w . \tag{39}$$

Any real nw-by-one vector w will yield a gain adjustment vector ΔK that satisfies the hard constraint equation. The second term in (39) represents the degrees of freedom allowed by the hard constraint equation. These degrees of freedom are used to minimize the extended quadratic approximation of the objective function. Use equation (39) to perform a change of variables and rewrite (31) as an explicit function of the vector w.

$$\hat{r}_E(K, w) = \sum_{i=1}^{ny} \alpha_i^2 \left\| P_{Wi}(t, K)\, V_{H0} w - \left(e_{Wi}(t, K) - P_{Wi}(t, K)\, \Delta K_{min} \right) \right\|^2 + \sum_{j=1}^{p} \beta_j^2 \left\| M_j\, V_{H0} w - \left(\bar{m}_j(K) - M_j\, \Delta K_{min} \right) \right\|^2 . \tag{40}$$

Equation (40) encompasses the original objective function as well as all the hard and soft gain constraints. It is a nonnegative definite, scalar, real-valued function of the vector w, and can be written in the quadratic form if the following two functions of K are defined:

$$Q_E(K) := V_{H0}^T [\, Q(K) + Q_S \,] V_{H0} , \tag{41}$$

$$b_E(K) := V_{H0}^T \left[\, b(K) + b_S(K) - (Q(K) + Q_S)\, \Delta K_{min} \right] . \tag{42}$$

Equations 41 and 42 are implemented in computational block 205 of FIG. 2 to calculate the nw-by-nw matrix $Q_E(K)$ and the nw-by-one vector $b_E(K)$. The quadratic form of the extended quadratic approximation of the objective function is given by

$$\hat{r}_E(K,w) = w^T Q_E(K) w - 2\, w^T b_E(K) + \hat{r}_E(K, \Delta K_{min}) . \tag{43}$$

The gradient of $\hat{r}_E(K,w)$ with respect to w is the column vector

$$\frac{\partial \hat{r}_E(K,w)}{\partial w} = 2\, Q_E(K) w - 2\, b_E(K) . \tag{44}$$

For a fixed gain vector K, the function $\hat{r}_E(K,w)$ is minimized when the vector w is chosen to make this gradient equal to zero. Thus $\hat{r}_E(K,w)$ will be minimized when the equation

$$Q_E(K) w = b_E(K) \tag{45}$$

is satisfied. Equation (45) is solved by performing a spectral decomposition of the symmetric nonnegative definite nw-by-nw matrix $Q_E(K)$.

$$Q_E(K) = V_E\, \Lambda_E\, V_E^T , \tag{46}$$

where Λ_E is an nw-by-nw diagonal matrix of the eigenvalues of $Q_E(K)$ arranged in decreasing order. The set of eigenvalues $\{\lambda_{E1}, \ldots, \lambda_{Enw}\}$ is called the spectrum of $Q_E(K)$. The matrix V_E

is an nw-by-nw orthogonal matrix. The columns of V_E are the eigenvectors of $Q_E(K)$, and form an orthonormal basis for nw-space. Partition these matrices such that

$$\Lambda_E = \begin{bmatrix} \Lambda_{E1} & 0 \\ 0 & 0 \end{bmatrix}, \quad V_E = [\, V_{E1} \;\; V_{E0} \,], \tag{47}$$

where Λ_{E1} is a diagonal matrix of the strictly positive eigenvalues of $Q_E(K)$. The pseudoinverse of $Q_E(K)$ is calculated as

$$Q_E^{+}(K) = V_{E1}\, \Lambda_{E1}^{-1}\, V_{E1}^{T}, \tag{48}$$

and the columns of V_{E0} form an orthonormal basis for the null space of $Q_E(K)$. The spectral decomposition of the matrix $Q_E(K)$ along with equations 47 and 48 are implemented in computational block **206** of FIG. **2** to calculate the nw-by-nw matrix $Q_E^{+}(K)$. The minimum two-norm solution to equation (45) is given by

$$w = Q_E^{+}(K)\, b_E(K). \tag{49}$$

Equation 49 is implemented in computational block **207** of FIG. **2** to calculate the nw-by-one vector w. Equation 39 is implemented in computational block **208** of FIG. **2** to calculate the nk-by-one vector of gain adjustments ΔK. Equation 10 is implemented in computational block **209** of FIG. **2** to calculate the predicted new process responses $\hat{y}(t, K, \Delta K)$.

FIG. **3** shows a flowchart describing a flow of steps to shape the time responses of a controlled process in accordance with the invention. The instructions necessary for the execution of the flow of steps are contained in read-only memory **112** of FIG. **1**. In step **0**, the desired process responses, gain constraints, and weights are defined and recorded in memory **111** of FIG. **1**. In step **1**, the process time responses are generated from controlled process **101** and stored in memory **107**. Also in step **1**, the response sensitivities are generated and stored in memory **108**. In step **2**, the gain adjustments and predicted new process responses are calculated according to the mathematical sequence shown in FIG. **2**. If the calculated gain adjustments and predicted new process responses are not satisfactory, then modify desired process responses, gain constraints, and weights and return to step **2**. When the calculated gain adjustments and predicted new process responses are satisfactory, then proceed to step **3** where tuneable gains **103** are modified to reflect calculated gain adjustments. In step **4**, the actual new process time responses are generated from controlled process **101**. If the actual new process time responses do not resemble the predictions, then one or more of the gain adjustments is too large. Modify desired process responses, gain constraints, or weights to limit the magnitudes of the offending gain adjustments and return to step **2**. If the actual new process time responses do resemble the predictions but do not yet satisfy the design requirements, then return to step **1** for another iteration of the response shaping process.

Figures 4 through 9 illustrate time response and gain space windows that aid in the definition and subsequent modification of the desired process responses, gain constraints, and weights. This method of graphical presentation of the data makes the manipulation of these quantities simple and intuitive. Information necessary to generate these figures is output by optimizing computer **110** in FIG. **1** for display on optional video display **114**.

FIG. **4** shows an example of a time response window. The window displays the current process time response, desired process time response, predicted new process time response, and weighting for one of the outputs of fixed portion of controlled process **102**.

The horizontal bar beneath the time responses and the two vertical bars to the left provide a convenient, graphical method of forming and modifying the weighting. The operator uses optional keyboard or mouse **113** in FIG. **1** to wipe the three shades of fill onto the horizontal bar, and also to set the vertical bars to desired levels. The weighting takes on the value set by the vertical bar of a certain shade everywhere that the horizontal bar is filled with that shade. The weighting is zero everywhere that the horizontal bar is not shaded. The example configuration of the vertical and horizontal bars results in the weighting shown in FIG. **4**.

The weighting is plotted along with the responses in FIG. **4** in order to aid in the description of the bars. The weighting is not displayed in the actual implementation of the response window. The signal labels are not shown either.

FIG. **5** shows an example of a gain space window. In this example, two lines and a point are displayed in the gain-space. The nominal gain vector, gain adjustment vector, and resulting tuned gain vector are also displayed. The squared Euclidean distance separating the tuned gain vector from each line or point is penalized at the level determined by the corresponding vertical bar to the left. An infinite value of beta results in a hard constraint, forcing the tuned gain vector onto the corresponding line or point. A finite value of beta results in a soft constraint, and a zero beta allows the solution to ignore the corresponding line or point.

The operator uses optional keyboard or mouse **113** in FIG. **1** to position the lines and points and to set the levels of the vertical bars. The actual implementation of the gain-space window does not label the gain vectors or lines and points, and does not explicitly show the gain adjustment vector.

FIG. **6** shows another example of a gain space window. Step **2** in the flowchart of FIG. **3** usually must be executed a number of times before the new gains and predicted new time responses are all satisfactory. The gain space window of FIG. **6** shows a history of the

tuning of a single gain during these iterations. A desired gain value and the weighting on that desire can also be shown in the window. The operator uses optional keyboard or mouse **113** in FIG. **1** to set the desired gain value and to set the level of the vertical bar.

FIG. **7** shows another example of a gain space window. It is often extremely illuminating to see the contours of constant value of the quadratic approximation of the objective function $\hat{r}(K, \Delta K)$ plotted in the gain space. Since $\hat{r}(K, \Delta K)$ is a quadratic function of the gains, the contours of constant value are concentric ellipsoids centered around the minimizing gain vector solution. The gain-space window displays a selected two-dimensional slice of the gain-space, and the contours of constant value can be shown as concentric ellipses. It is important to know if there are multiple gain solutions which would result in equally (or almost equally) good predicted new process responses. The example window of FIG. **7** shows that deviation from the optimal solution in the SD2 direction has relatively little adverse effect on the predicted new process responses.

FIG. **8** shows another example of a gain space window. The value of the quadratic approximation of the objective function and the gain values are plotted along the direction SD1. Note that the curvature of $\hat{r}(K, \Delta K)$ is severe along this direction. Any deviation from the optimal solution in direction SD1 results in a severe degradation of the quality of the predicted new process responses.

FIG. **9** shows another example of a gain space window. The value of the quadratic approximation of the objective function and the gain values are plotted along the direction SD2. Note that the curvature of $\hat{r}(K, \Delta K)$ is mild along this direction. Deviation from the optimal solution in the SD2 direction has relatively little adverse effect on the quality of the predicted new process responses.

FIG. **10** illustrates a complete display that would be presented on optional video display **114** of FIG. **1**. The display includes two time response windows and five gain space windows. In the example, the arm position response and motor current response of a servo are plotted in two time response windows. Two tuneable gains, KI and KP are used to shape these time responses. The tuning history of each gain is plotted in two gain space windows. A third gain space window shows the contours of constant value of the objective function in the KI-KP space. A fourth gain space window shows the value of the objective function and the gain values along a selected direction in the KI-KP space.

While the invention has been described with reference to a preferred embodiment, it will be understood by those skilled in the art that various changes may be made and equivalents may be substituted for elements thereof without departing from the scope of the invention. In

addition, many modifications may be made to adapt a particular situation to the teachings of the invention without departing from the essential scope thereof. Therefore, it is intended that the invention not be limited to the particular embodiment disclosed as the best mode contemplated for carrying out this invention, but that the invention will include all embodiments falling within the scope of the appended claims.

What is claimed is:

1. A method for the shaping of time responses of a multi-input, multi-output controlled physical process, the shaping method comprising the steps of:

storing desired physical process responses in read only memory,

generating a series of experiments on the physical process to generate current physical process responses,

storing the current physical process responses in random access memory,

generating and storing response sensitivities for the current physical process responses by the method of Variable Components,

calculating new gain values by quadratic optimization,

predicting new process responses, and

applying the new gain values to the controlled physical process.

2. The method according to claim 1 wherein said quadratic optimization procedure comprises the steps of

(a) calculating the matrices that comprise the quadratic forms of an approximate objective function and a soft constraint objective function,

(b) appending hard constraint functions to form a single hard constraint equation and performing a singular value decomposition to calculate the minimum two-norm solution to a hard constraint equation,

(c) calculating the matrices that comprise the quadratic form of an extended objective function and performing a spectral decomposition to calculate the minimum two-norm solution that minimizes the extended objective function, and

(d) combining the solutions to (b) and (c) to form the gain adjustment vector and using the physical process responses and response sensitivities to calculate a first-order Taylor's series approximation of new physical process responses.

3. The method according to claim 1 wherein said quadratic optimization procedure comprises the step of displaying graphical time response windows for the purpose of displaying actual, desired, and predicted process time responses and adjustable response weighting bars.

4. The method according to claim 1 wherein said quadratic optimization procedure comprises the step of displaying graphical gain space windows for the purpose of displaying gain values, contours of the constant value of the objective function, gain constraints, and adjustable constraint weighting bars.

5. Optimizing computer apparatus for the shaping of time responses of a multi-input, multi-output controlled physical process, the apparatus comprising

a read only memory for storing desired physical process responses,

data extraction means for extracting process responses and response sensitivities from the controlled physical process,

control means, responsive to the read only memory and the data extraction means, for controlling the physical process by quadratic optimization and calculating gain values, and

application means for applying the gain values to the physical process.

6. Apparatus according to claim 5 wherein the data extraction means comprises inputs and sensors and the control means comprises outputs for at least the purpose of obtaining response sensitivity signals.

7. Apparatus according to claim 6 further comprising data-path switches, responsive to the sensors, for the purpose of obtaining process time responses and the response sensitivity signals.

8. Apparatus according to claim 6 wherein said apparatus comprises random access memory coupled to the outputs of the control means for recording process time responses, response sensitivity signals, and sensitivity excitation signals.

9. Apparatus according to claim 5 wherein said control means reads and writes data to memory for storing desired process responses, gain constraints, and weights.

10. Apparatus according to claim 5 further comprising input means for adjusting tuneable gains of said controlled physical process and display means for displaying windows into the controlled physical process.

11. Apparatus according to claim 10, the windows particularly comprising gain space windows.

12. Apparatus according to claim 10, the windows particularly comprising time response windows.

13. Apparatus according to claim 10, the input means particularly comprising a mouse.

14. A method for shaping time responses of a multi-input, multi-output controlled physical process, the shaping method comprising the steps of

extracting data from the physical process, the data representing response sensitivity signals,

storing desired response data in read only memory,

calculating new gain values for the process by means of quadratic optimization, and

applying the new gain values to the physical process.

15. The method according to claim 14, wherein the data extraction step employs the method of Variable Components.

16. The method according to claim 14, wherein the data extraction step employs the method of tuning sensitivity points.

17. The method according to claim 14 further comprising the step of displaying windows to the physical process.

18. The method according to claim 17, the windows particularly comprising time response windows.

19. The method according to claim 17, the windows particularly comprising gain space windows.

20. The method according to claim 14 further comprising the step of enabling adjustment of responses and constraints by means of a mouse.

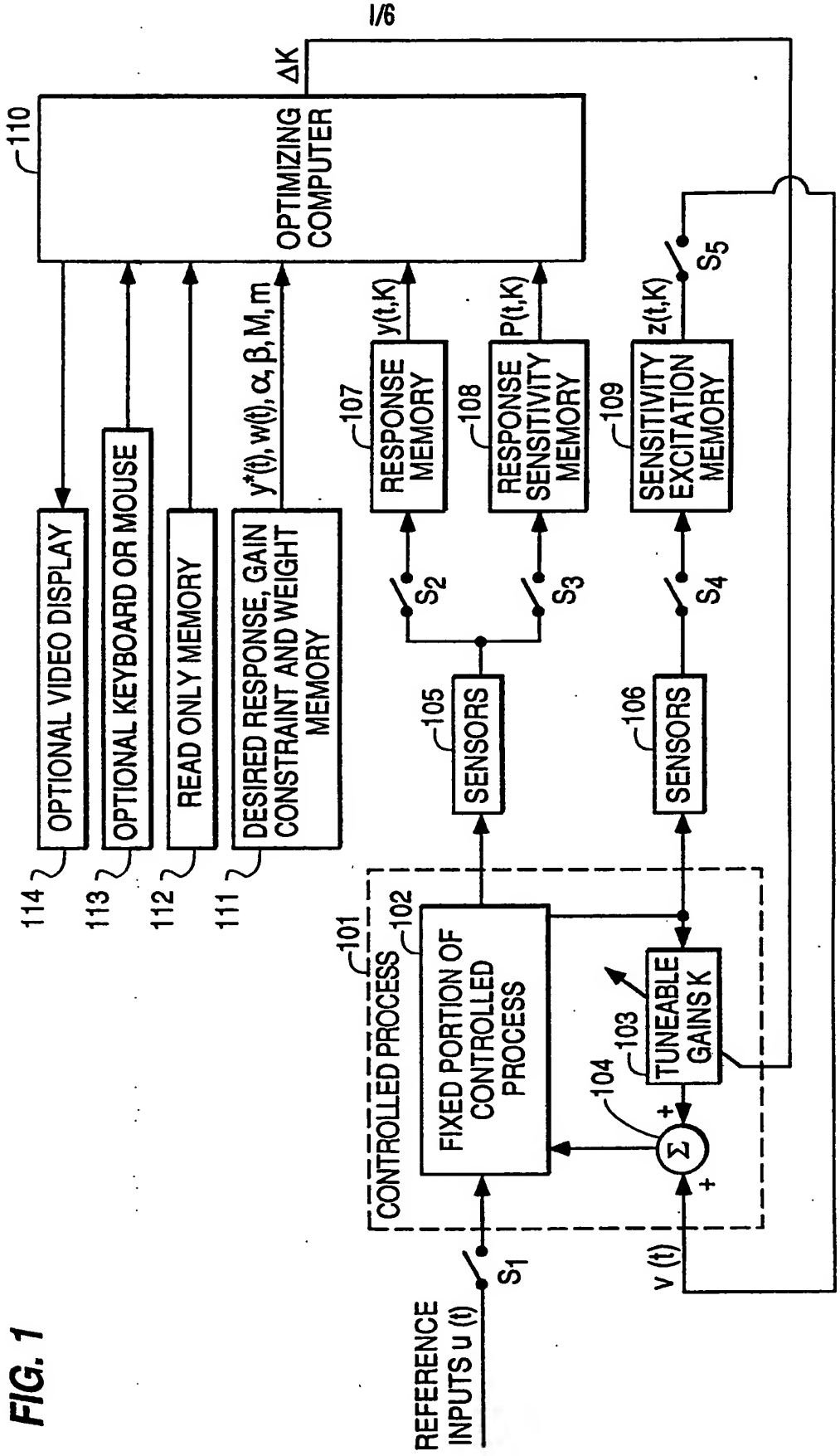
ΔK
110
OPTIMIZING COMPUTER
OPTIONAL VIDEO DISPLAY
OPTIONAL KEYBOARD OR MOUSE
READ ONLY MEMORY
DESIRED RESPONSE, GAIN CONSTRAINT AND WEIGHT MEMORY
y*(t), w(t), α, β, M, m
114
113
112
111
y(t,K)
P(t,K)
z(t,K)
RESPONSE MEMORY
RESPONSE SENSITIVITY MEMORY
SENSITIVITY EXCITATION MEMORY
107
108
109
S2
S3
S4
S5
SENSORS
SENSORS
105
106
CONTROLLED PROCESS
101
FIXED PORTION OF CONTROLLED PROCESS
102
TUNEABLE GAINS K
103
104
Σ
+
+
S1
REFERENCE INPUTS u (t)
v (t)

FIG. 1

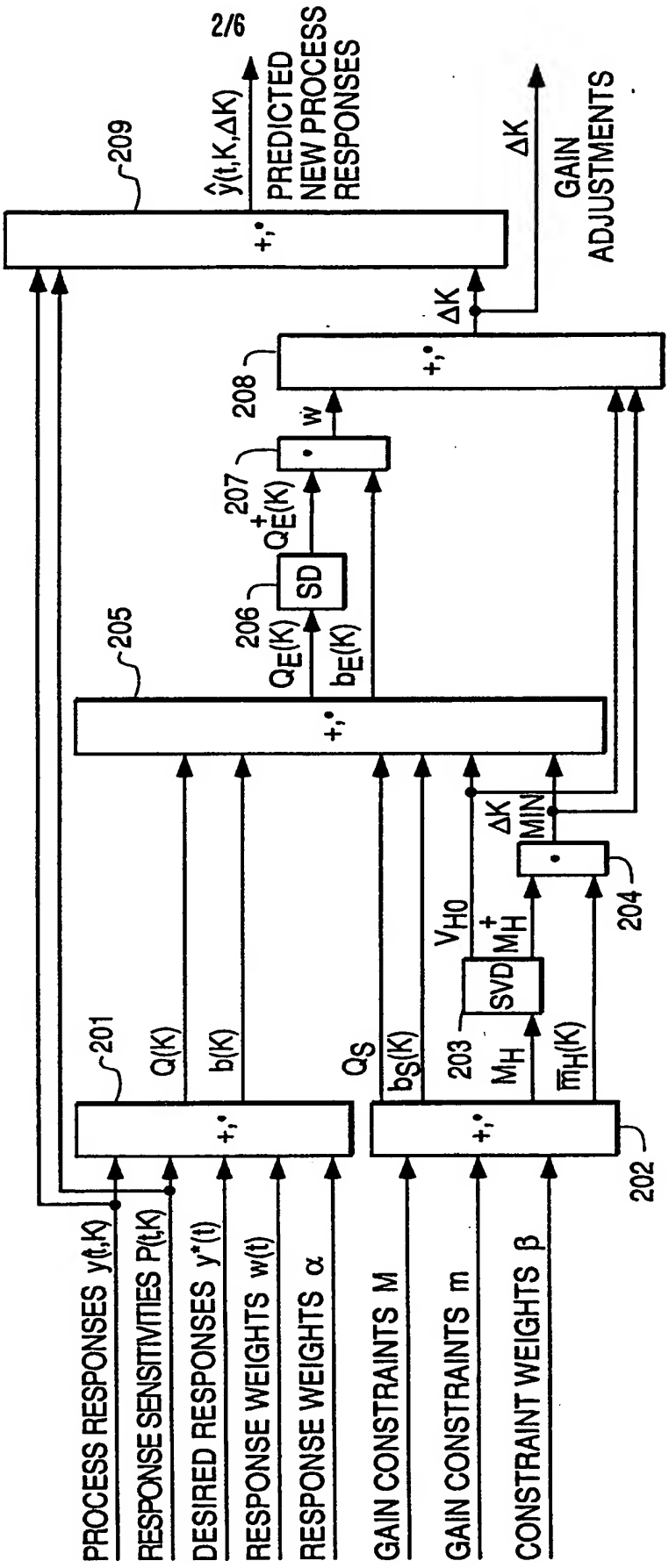
PROCESS RESPONSES y(t,K)
RESPONSE SENSITIVITIES P(t,K)
DESIRED RESPONSES y*(t)
RESPONSE WEIGHTS w(t)
RESPONSE WEIGHTS α
GAIN CONSTRAINTS M
GAIN CONSTRAINTS m
CONSTRAINT WEIGHTS β
201
Q(K)
b(K)
Q_S
b_S(K)
203
M_H
SVD
V_H0
M_H+
ΔK
MIN
204
m_H(K)
202
205
Q_E(K)
206
SD
Q_E+(K)
207
b_E(K)
208
w
209
ŷ(t,K,ΔK)
PREDICTED NEW PROCESS RESPONSES
ΔK
GAIN ADJUSTMENTS

FIG. 2

FIG. 3

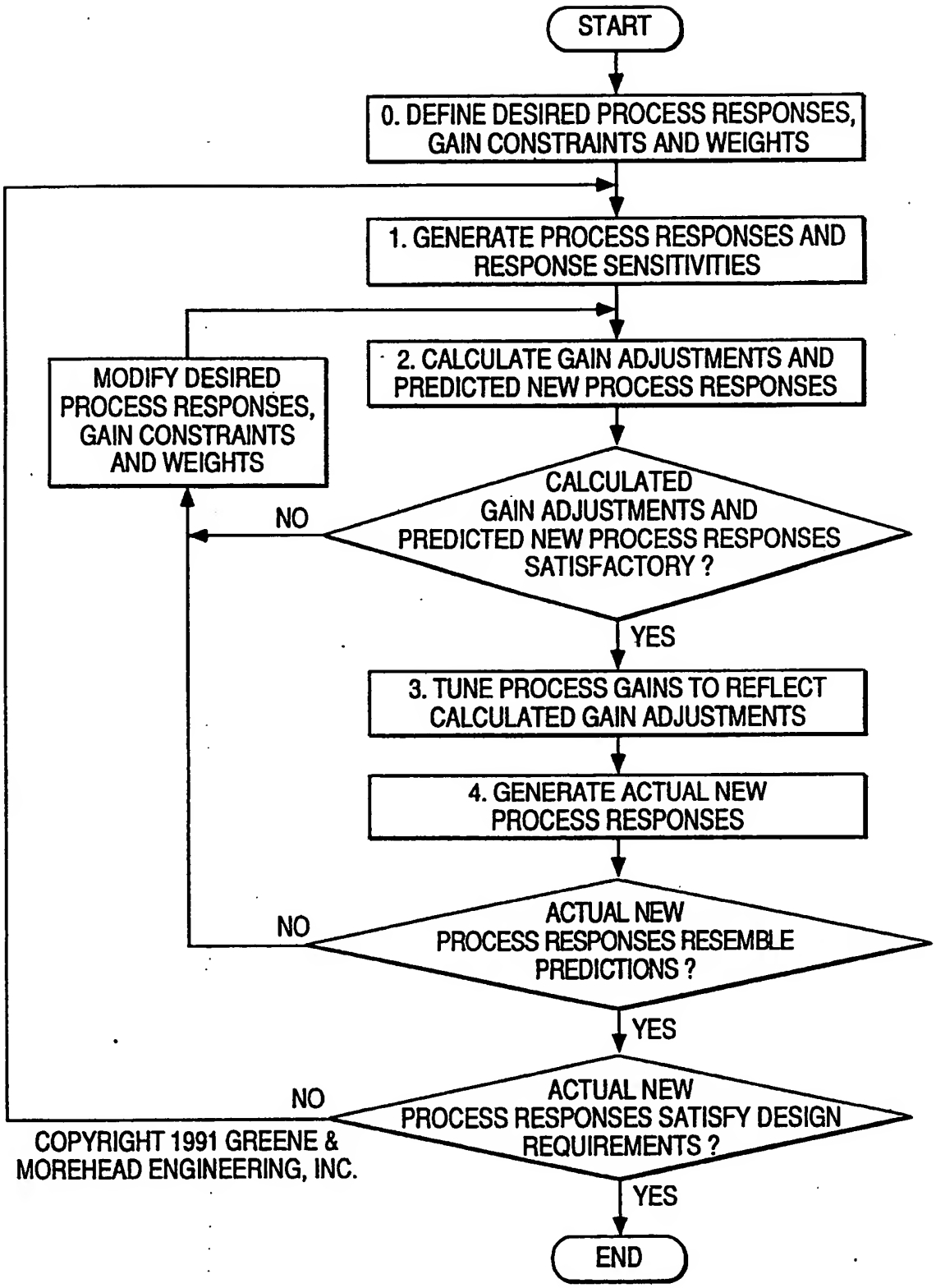

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FIG. 4

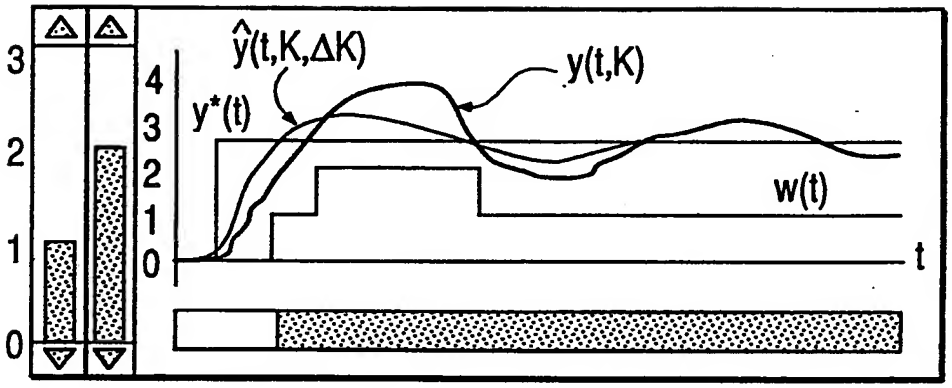


FIG. 5

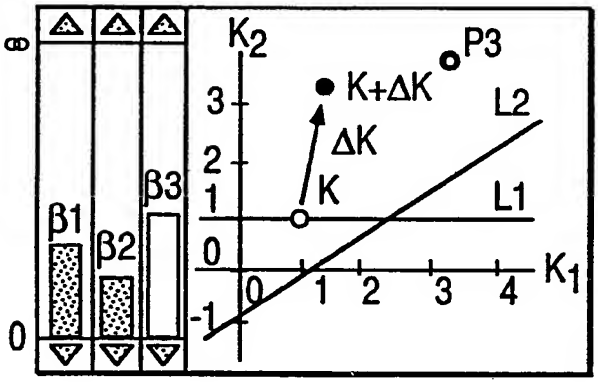


FIG. 6

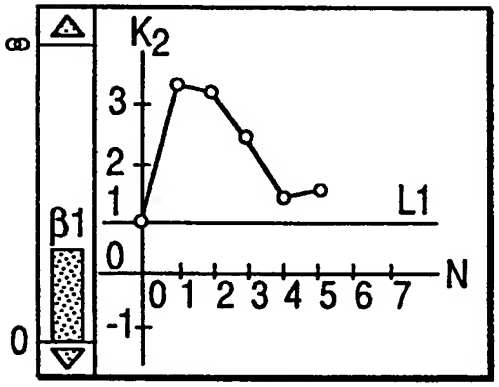


FIG. 7

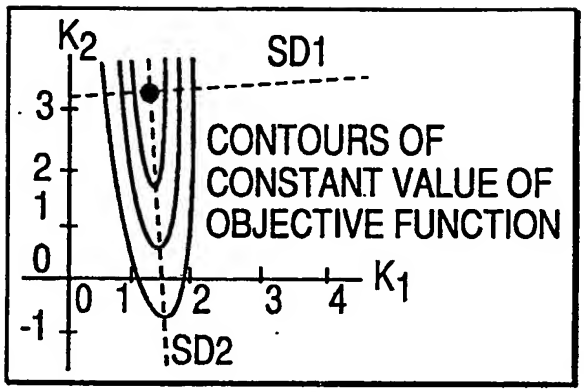


FIG. 8

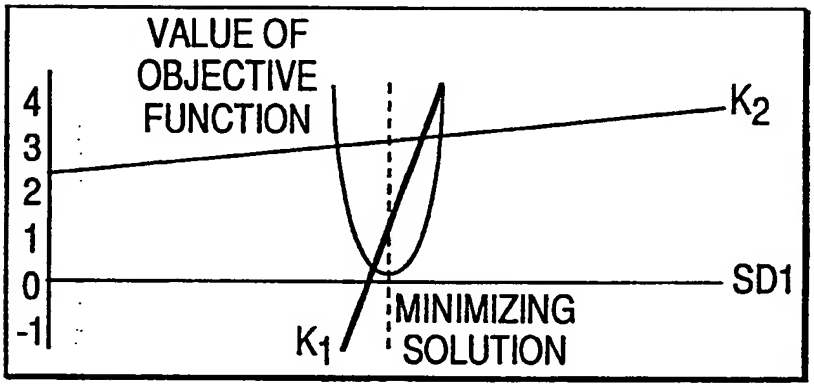
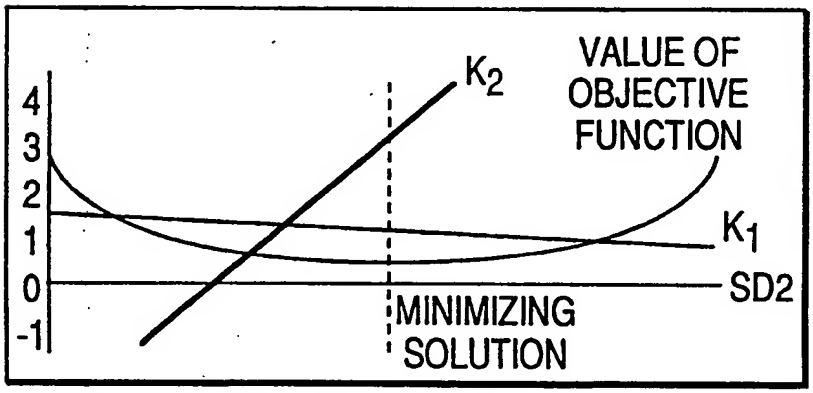


FIG. 9



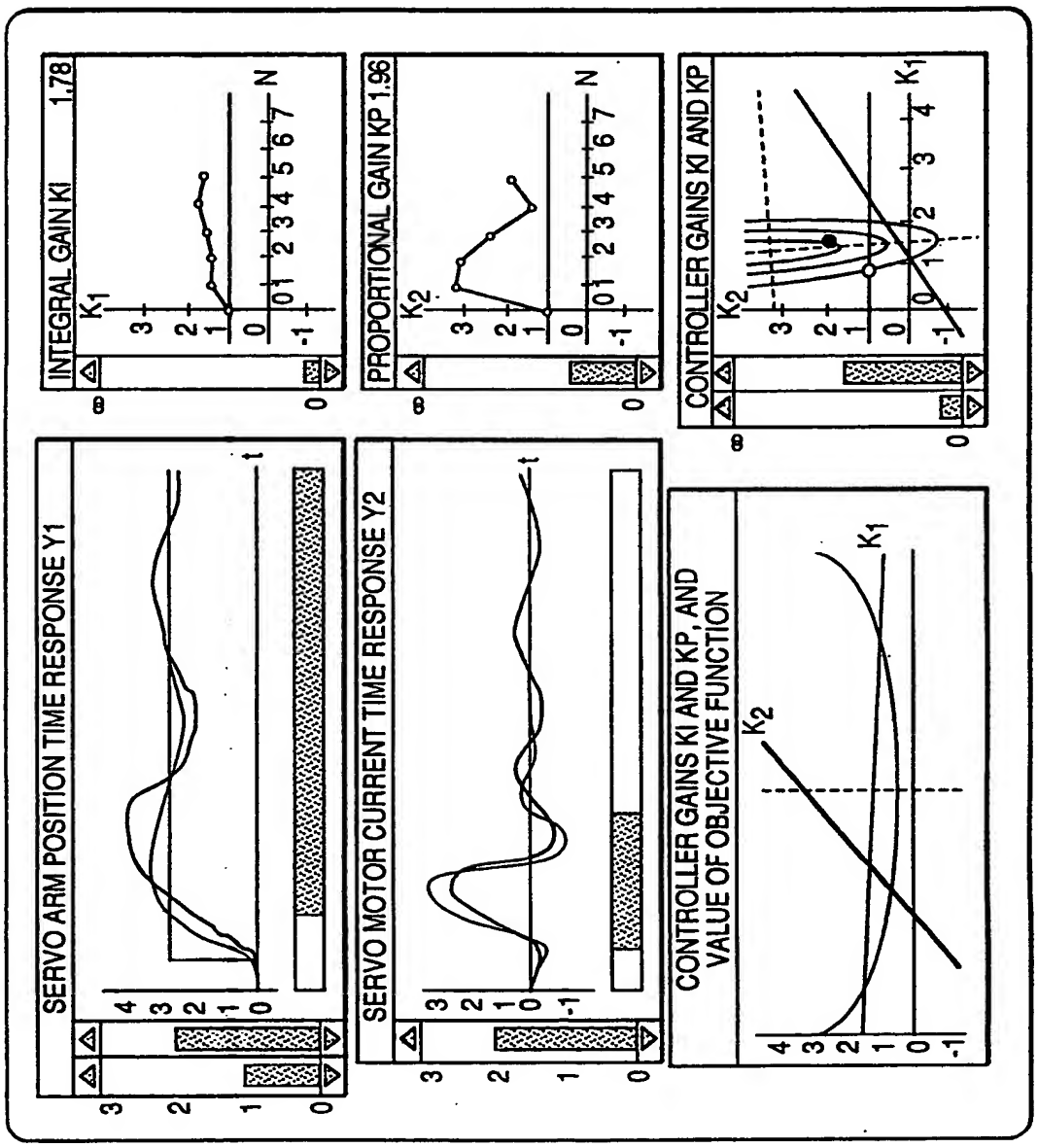

SERVO ARM POSITION TIME RESPONSE Y1
SERVO MOTOR CURRENT TIME RESPONSE Y2
INTEGRAL GAIN KI 1.78
PROPORTIONAL GAIN KP 1.96
CONTROLLER GAINS KI AND KP
CONTROLLER GAINS KI AND KP, AND VALUE OF OBJECTIVE FUNCTION
K1
K2
N
t
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FIG. 10

INTERNATIONAL SEARCH REPORT

PCT/US 92/04427

International Application No.

I. CLASSIFICATION OF SUBJECT MATTER (If several classification symbols apply, indicate all)[6]

According to International Patent Classification (IPC) or to both National Classification and IPC

Int.Cl. 5 G05B13/02

II. FIELDS SEARCHED

Minimum Documentation Searched[7]

Classification System	Classification Symbols
Int.Cl. 5	G05B

Documentation Searched other than Minimum Documentation to the Extent that such Documents are Included in the Fields Searched[8]

III. DOCUMENTS CONSIDERED TO BE RELEVANT[9]

Category[10]	Citation of Document,[11] with indication, where appropriate, of the relevant passages[12]	Relevant to Claim No.[13]
A	PATENT ABSTRACTS OF JAPAN vol. 10, no. 76 (P-440)(2133) 26 March 1986 & JP,A,60 215 208 (TATEISHI DENKI K.K.) 28 October 1985 see abstract	1,5,14
A	PROCEEDINGS OF THE 27TH IEEE CONFERENCE ON DECISION AND CONTROL vol. 3, December 1988, AUSTIN US pages 1947 - 1951 MAURICIO ALVES DA SILVA ET AL. 'A RULE BASED PROCEDURE FOR SELFTUNING PID CONTROLLERS' see paragraph 4; figure 5	1,3,4,5, 10,12, 14,17,18

* Special categories of cited documents :[10]

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

IV. CERTIFICATION

Date of the Actual Completion of the International Search	Date of Mailing of this International Search Report
23 SEPTEMBER 1992	01. 10. 92
International Searching Authority	**Signature of Authorized Officer**
EUROPEAN PATENT OFFICE	GOETZ P.A.

Form PCT/ISA/210 (second sheet) (January 1985)